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Modeling and simulations of 3-D turbulence wind field based on rapid distortion theory

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SUMMARY:
Major engineering structures, such as long-span bridges, high-voltage DC transmission tower line systems and floating offshore wind turbines (FOWTs), are typical three-dimensional (3-D) wind-sensitive structures, and wind-induced vibrations or even damages of these structures are usually caused by the coupled effects of 3-D wind loads. In this view, the 3-D turbulence wind field should be considered in the wind-induced response analysis and wind-resistant reliability assessment of complex structures. In this paper, a physical 3-D turbulence wind field modeling approach based on rapid distortion theory (RDT) is proposed. From the basic equations of fluid mechanics, the linearized governing equations of homogenous turbulence are firstly deduced by introducing RDT. Then, the shear spectral tensor of atmospheric surface-layer turbulence field is derived based on the spectral analysis theory and the spectral tensor of isotropic turbulence. On this basis, the 3-D turbulence wind field can be simulated by means of spectral decomposition techniques. In order to circumvent the dilemma of high-dimensional random variables in the numerical simulation formula, the evolutionary phase model (EPM) is introduced to represent the random phase angles with just 4 elementary random variables, and the Fast Fourier Transform algorithm is also used to enhance the efficiency of numerical simulations. Using the proposed method, a turbulence wind field of an offshore wind turbine is simulated. The numerical results indicate that the proposed method can reflect the physical mechanism of the atmospheric boundary layer shear turbulence field.

Keywords: 3-D turbulence wind field, rapid distortion theory, shear spectral tensor, evolutionary phase model.

1. SHEAR SPECTRAL TENSOR BASED ON RDT

Let $\mathbf{v}(\mathbf{x},t)=[v_1(\mathbf{x},t),v_2(\mathbf{x},t),v_3(\mathbf{x},t)]$ be the turbulence wind speed vector with its elements $v_1(\mathbf{x},t),v_2(\mathbf{x},t)$ and $v_3(\mathbf{x},t)$ are the turbulence components corresponding to the longitudinal x_1, lateral x_2 and vertical x_3, respectively. The turbulence component $v_j(\mathbf{x},t),\ (j=1,2,3)$ is a zero-mean Gaussian random process, which can be expressed as the following Fourier-Stieltjes integral (Lumley, 1970)

$$v_j(\mathbf{x},t)=\int_{R^3}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{x})\mathrm{d}Z_j(\mathbf{k},t) \tag{1}$$

where $\mathbf{k}$ is the shear wave number; $\mathrm{d}Z_j(\mathbf{k},t)$ satisfies the following conditions

$$\mathrm{E}\left[\mathrm{d}Z_i(\mathbf{k},t)\mathrm{d}Z_j^*(\mathbf{k},t)\right]=\Phi_{ij}(\mathbf{k},t)\mathrm{d}\mathbf{k} \tag{2}$$

In which $\Phi_{ij}(\mathbf{k},t)$ is the shear spectral tensor of the turbulence. In general, the physical turbulence spectral tensor is determined by the turbulence model, of which the expression is derived through the physical governing equation, i.e. Navier-Stokes equation, of the turbulence field. It is acknowledged that the Navier-Stokes equation does not have the closed-form solutions. However, as for the atmospheric boundary layer shear turbulence field, the Navier-Stokes equation can be transformed into a linearized ODE form by introducing the RDT approximation, and the shear spectral tensor can be thus derived from the spectral tensor of the isotropic turbulence. The components of the shear spectral tensor can be expressed as (Mann, 1994)

$$\Phi_{11}(k)=\frac{E(k_0)}{4\pi k_0^4}\left(k_{0,3}^2+k_2^2-2k_1k_{0,3}\zeta_1+(k_1^2+k_2^2)\zeta_1^2\right) \tag{3}$$

$$\Phi_{22}(k)=\frac{E(k_0)}{4\pi k_0^4}\left(k_1^2+k_{0,3}^2-2k_2k_{0,3}\zeta_2+(k_1^2+k_2^2)\zeta_2^2\right) \tag{4}$$

$$\Phi_{33}(k)=\frac{E(k_0)}{4\pi k^4}(k_1^2+k_2^2) \tag{5}$$

$$\Phi_{12}(k)=\Phi_{21}(k)=\frac{E(k_0)}{4\pi k_0^4}\left(-k_1k_2-k_1k_{0,3}\zeta_2-k_2k_{0,3}\zeta_1+(k_1^2+k_2^2)\zeta_1\zeta_2\right) \tag{6}$$

$$\Phi_{13}(k)=\Phi_{31}(k)=\frac{E(k_0)}{4\pi k_0^2k^2}\left(-k_1k_{0,3}+(k_1^2+k_2^2)\zeta_1\right) \tag{7}$$

$$\Phi_{23}(k)=\Phi_{32}(k)=\frac{E(k_0)}{4\pi k_0^2k^2}\left(-k_2k_{0,3}+(k_1^2+k_2^2)\zeta_2\right) \tag{8}$$

where $k_0=\sqrt{k_{0,1}^2+k_{0,2}^2+k_{0,3}^2}$ and $k=\sqrt{k_1^2+k_2^2+k_3^2}$ are the magnitudes of the initial wave number and the shear wave number, respectively; $E(k_0)$ is the scaler energy spectrum function of the isotropic turbulence, and the von Karman energy spectrum model is selected in this paper, of which the expression is given as follows (Batchelor, 1953):

$$E(k_0)=\varepsilon\eta^{2/3}L^{5/3}\frac{(Lk_0)^4}{(1+(Lk_0)^2)^{17/6}} \tag{9}$$

where ε is the Kolmogorov constant; η is the rate of viscous dissipation of specific turbulent kinetic energy; L is the length scale of the isotropic turbulence. The expressions of $\zeta_1(t)$ and $\zeta_2(t)$ in Eqs. (3) - (8) are given by Mann (1994).

2. SIMULATION FORMULA BASED ON THE EVOLUTIONARY PHASE MODEL

2.1. Simulation Formula

The infinite integral in Eq. (1) is approximated as follows by the spectral decomposition

$$\begin{aligned}v_j(x_{l_1},x_{l_2},x_{l_3})\approx 2\mathrm{Re}\sum_{n_1=1}^{N_1}\sum_{n_2=1}^{N_2}\sum_{n_3=1}^{N_3}\Big\{&\exp[\mathrm{i}(x_{l_1}k_{n_1}+x_{l_2}k_{n_2}+x_{l_3}k_{n_3})]\Delta Z_j(k_{n_1},k_{n_2},k_{n_3})+\\&\exp[\mathrm{i}(-x_{l_1}k_{n_1}+x_{l_2}k_{n_2}+x_{l_3}k_{n_3})]\Delta Z_j(-k_{n_1},k_{n_2},k_{n_3})+\\&\exp[\mathrm{i}(x_{l_1}k_{n_1}-x_{l_2}k_{n_2}+x_{l_3}k_{n_3})]\Delta Z_j(k_{n_1},-k_{n_2},k_{n_3})+\\&\exp[\mathrm{i}(x_{l_1}k_{n_1}+x_{l_2}k_{n_2}-x_{l_3}k_{n_3})]\Delta Z_j(k_{n_1},k_{n_2},-k_{n_3})\Big\}\end{aligned} \tag{10}$$

where

$$\begin{cases}\Delta Z_j(-k_{n_1},k_{n_2},k_{n_3})=B_{jl}(\tau)H_{lm}(-k_{0,n_1},k_{0,n_2},k_{0,n_3})\sqrt{\Delta k_0}\exp[\mathrm{i}\Theta_m^{(2)}(\mathbf{k}_0)]\\\Delta Z_j(k_{n_1},-k_{n_2},k_{n_3})=B_{jl}(\tau)H_{lm}(k_{0,n_1},-k_{0,n_2},k_{0,n_3})\sqrt{\Delta k_0}\exp[\mathrm{i}\Theta_m^{(3)}(\mathbf{k}_0)]\\\Delta Z_j(k_{n_1},k_{n_2},-k_{n_3})=B_{jl}(\tau)H_{lm}(k_{0,n_1},k_{0,n_2},-k_{0,n_3})\sqrt{\Delta k_0}\exp[\mathrm{i}\Theta_m^{(4)}(\mathbf{k}_0)]\end{cases} \tag{11}$$

in which $\Theta_m^{(r)}(\mathbf{k}_0)$'s, $(r=1,2,3,4)$ are the identically distributed random variables uniformly distributed over the interval $[0,2\pi]$; $H_{lm}(\cdot)$ is the element of the lower triangular matrix $\mathbf{H}(\cdot)$ which is the Cholesky decomposition of $\Phi_{ij}(\mathbf{k},t)$; $B_{jl}(\tau)$ is the element of $\mathbf{B}(\tau)$ matrix, of which the expression can be referred to Mann (1994).

2.2. Random Variables Based on the Evolutionary Phase Model

It is acknowledged from the above section that the number of random variables is $4\times 3\times N_1\times N_2\times N_3$ if one wants to generate a 3-D turbulence wind field by using the Monte Carlo method. This is quite time-consuming in engineering practice. In view of this, the EPM (Yan and Li, 2011) is adopted in this paper to generate the phase variables. By using the EPM, the phase variables can be expressed as follows

$$\Theta_m^{(r)}(\mathbf{k}_0)=\mathrm{mod}(|\mathbf{k}_0|\Delta\sigma_m(\mathbf{k}_0)T_e^{(r)},2\pi) \tag{12}$$

where $\mathrm{mod}(\cdot)$ denotes the congruence symbol; $\Delta\sigma_m(\mathbf{k}_0)$ is the characteristic velocity for Fourier components of the turbulence in the *m*-direction with wave number vectors in $\Delta\mathbf{k}_0$, of which the expression is given as follows

$$\Delta\sigma_m^2(\mathbf{k}_0)=\Phi_{mm}(\mathbf{k}_0)\Delta k_0 \tag{13}$$

In Eq. (12), $T_e^{(r)}(r=1,2,3,4)$ is the evolution time of the evolutionary phases which satisfies the following Gamma distribution

$$f_{T_e}(t_e)=t_e^{(\beta-1)}\frac{\mathrm{e}^{-t_e/\theta}}{\theta^{\beta}\Gamma(\beta)} \tag{14}$$

where β and θ are the shape parameter and the scale parameter of the Gamma distribution, respectively. According to Li et al., (2013), $\beta=1.1$, $\theta=8.2\times 10\,\hat{}\,8$.

It can be seen that the number of random variables in Eq. (10) is greatly reduced to 4 after introducing the EPM, which significantly reduce the difficulty of random simulation for 3-D turbulent fields. Besides, the three-dimensional FFT algorithm can also be employed in the proposed approach to enhance the computational efficiency for numerical generating sample processes.

3. NUMERICAL EXAMPLE AND RESULTS

In order to illustrate the proposed approach, a numerical 3-D turbulence wind field for the 5-MW Hywind FOWT will be simulated. According to the structural parameters of the FOWT, wind field parameters of the numerical example are listed in Table 1.

Table 1. Wind field parameters of the numerical example

Parameters	Values	Parameters	Values
Time interval d*t* (s)	0.25	Vertical-wind spatial length L_3 (m)	8*L*
Mean wind speed at hub (m/s)	10	Spatial discrete points N_1	1024
Along-wind spatial interval d*x* (m)	2.5	Spatial discrete points N_2	32
Simulation duration (s)	256	Spatial discrete points N_3	32
Characteristic length scale *L* (m)	63	Dimensionless parameter τ_0	3.9
Along-wind spatial length L_1 (m)	2560	Von Karman energy parameter $\varepsilon\eta^{2/3}$	0.11
Across-wind spatial length L_2 (m)	8*L*	Mean shear rate α	0.14

Figure 1 shows the simulation results of the 3-D shear turbulence field by the proposed approach. It can be seen that the simulated turbulence components have the typical stationarity and randomness of the turbulence wind speed processes in the time domain, and the relationship of variance between different components is consistent with actual atmospheric observations, i.e. $\sigma_u^2 > \sigma_v^2 > \sigma_w^2$. In frequency domain, it can be seen that the simulated single-point PSDs are in good agreement with the target ones. In particular, the cross PSD between turbulence components u and w is negative, which reflects the energy transition between these two components in the shear turbulence field.

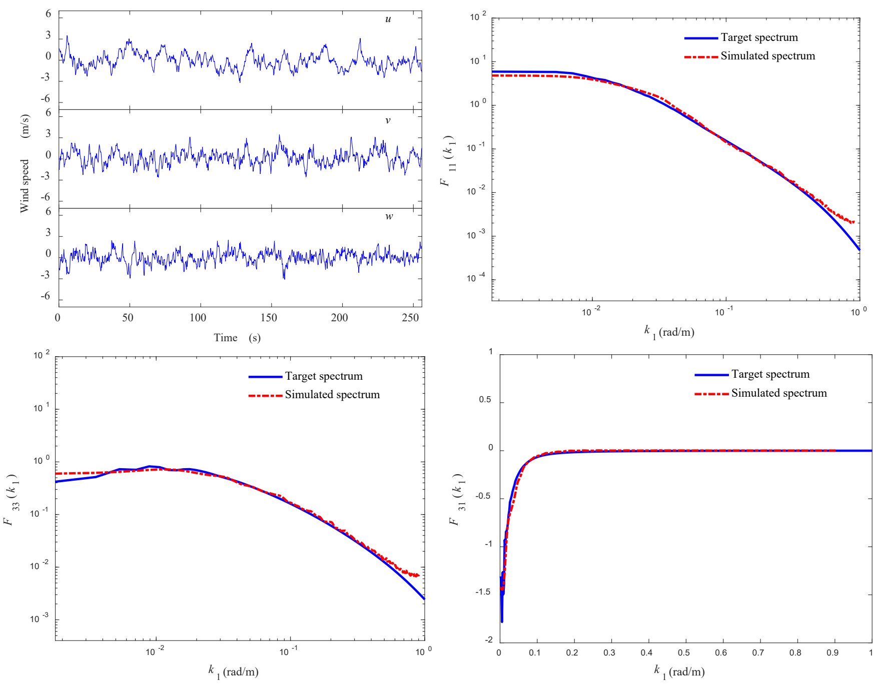

Figure 1. Simulation results of the shear turbulence wind velocity

4. CONCLUSIONS

A physical 3-D shear turbulence wind field modeling approach has been proposed in this paper. The physical shear spectral tensor model was firstly introduced on the basis of RDT. Then the discrete simulation formula was proposed for the numerical generation of the sample processes, and the ERM was employed to reduce the dimension of the random phase angles. Finally, a numerical example was carried out to simulate the 3-D turbulence wind velocities of a FOWT, the results indicate that the proposed approach can reasonably reflect the characteristics of the atmospheric boundary layer turbulence wind field in both time domain and frequency domain.

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